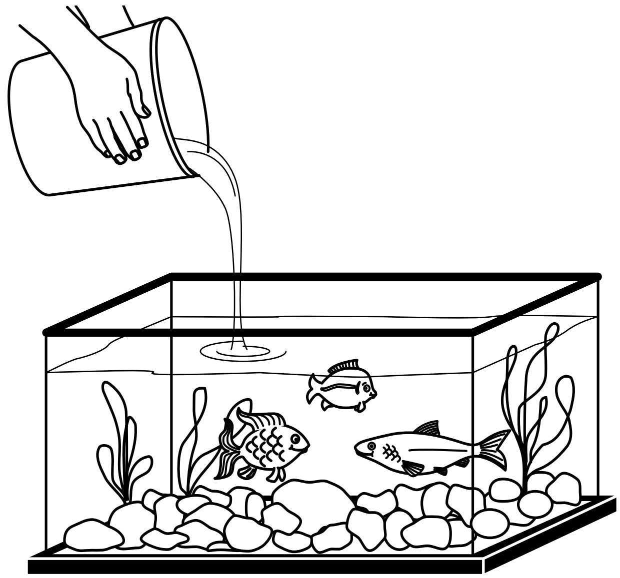
Investigation 8: How Do You Find the Volumes of Prisms and Cylinders?

Volume has applications in many different fields, especially science, engineering, architecture, and construction. It is also useful in everyday life. For example, how many buckets of water are needed to fill an aquarium like that shown at the right? In this activity, you will determine the volumes of various prisms and cylinders, and explore the relationships among their formulas.



How many 1-gallon buckets are needed to fill a fish tank?

Topic of Investigation

Influences of dimensions of prisms and cylinders on volume

Questions to Investigate

- *How do you find the volume of prisms and cylinders?*
- *How do the dimensions of a solid affect its volume?*
- *How does the formula for the volume of a cylinder relate to the formula for the volume of a prism?*

Mathematical Review

Volume the capacity of a solid or the amount of space the solid occupies. In this investigation, you will find the volumes of different prisms and cylinders.

Prism a three-dimensional figure with two parallel, congruent polygons as bases and at least three rectangular faces.

Cylinder a three-dimensional figure similar to a prism, except its bases are congruent circles instead of polygons. Thus, a cylinder does not have lateral faces.

To determine the volumes of various prisms and cylinders, you will calculate the areas of the solids' bases. Some of the area formulas you may need are shown below.

Area of a circle $= \pi r^2$ ($\pi \cong 3.14$ and r = radius)

Area of a square $= s^2$ (s = length of a side)

Area of a rectangle $= bh$ (b = length of the base and h = height)

Area of a triangle $= \frac{1}{2}bh$ (b = length of the base and h = height)

Predict the Results

Look at each of the solids listed in the table below. Without doing any calculations, predict how the volumes of these solids compare. Rank them according to volume, with 1 representing the greatest volume and 7 representing the least.

Predictions Chart: Volumes and Rankings

Relational GeoSolids Shape	Rankings by Volume (1 = greatest 7 = least)
Square Prism	
Rectangular Prism	
Small Triangular Prism	
Large Triangular Prism	
Small Cylinder	
Large Cylinder	
Large Cube	

Procedure

1. Using your ruler, measure the height of each geometric solid listed in the Results Chart to the nearest $\frac{1}{8}$ inch. Then determine the area of the base of the solid. Record your answers in the first and second columns.
2. Multiply the values in the first and second columns to determine the volume of each solid. Record your answers in the third column.
3. Rank the solids according to their volumes, with 1 representing the greatest volume and 7 representing the least. Record the rankings in the fourth column.

Materials

For each pair of students:

- *1 Relational GeoSolids set*
- *A customary or metric ruler*
- *A calculator*

Results Chart: Actual Volumes and Rankings

Relational GeoSolids Shape	Height (inches)	Area of Base (inches^2)	Volume (inches^3)	Rankings by Volume (1 = greatest 7 = least)
Square Prism				
Rectangular Prism				
Small Triangular Prism				
Large Triangular Prism				
Small Cylinder				
Large Cylinder				
Large Cube				

Discussion Questions

1. Compare your predictions with your results.

a. Do they match? If not, how are they different?

b. Explain why your predictions differ from the results.

c. How did your estimates compare with your classmates' estimates?

2. Look at your Results Chart.

a. In one sentence, summarize the measurements and the method you used to calculate the volumes of the solids.

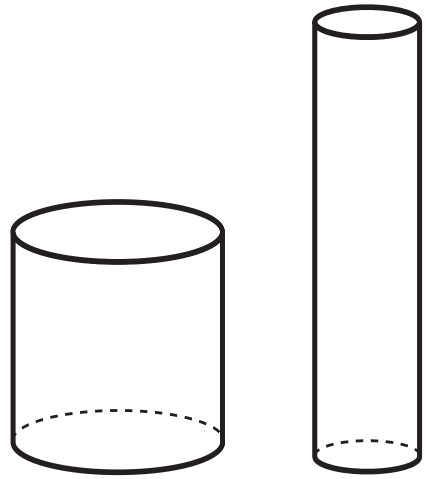
b. Why does this method for calculating volume work?

c. Do you think you could find the volume of any prism or cylinder by using this method? Explain your thinking.

3. The formula for the volume of a cylinder is $V = \pi r^2 h$. How is this formula related to the method you used to determine the volumes of the cylinders and prisms?

4. Cylinder X is twice as tall as the large Relational GeoSolids cylinder, but its radius is only half as long.

a. Which cylinder has the larger volume, Cylinder X or the large Relational GeoSolids cylinder? Explain.



b. How tall would Cylinder X have to be in order to have the same volume as the large Relational GeoSolids cylinder? (Hint: Volume of Cylinder X = Volume of large cylinder. Solve for the height (*h*) of Cylinder X.) Explain.

c. Which would have a greater effect on the volume of a cylinder—doubling its height or doubling the radius of its base? Explain.

Further Investigations

1. A rectangular box has three dimensions, length (l), width (w), and height (h).

a. What would be the effect on the volume of the box if l was doubled, but the other dimensions remained the same?

b. What would be the effect on the volume of the box if l and w were each doubled but h remained the same?

c. What would be the effect on the volume of the box if l, w, and h were all doubled?

d. What would be the effect on the volume of the box if l, w, and h were all tripled?

2. Determine the volume of the hexagonal prism using the same method you used for the other prisms. Explain all the steps you use to get your answer.

3. The science lab has a 36" wide × 18" deep × 20" high rectangular aquarium that needs to be filled. The students have a 12"-high cylindrical bucket with a 12"-diameter to fill the tank. If the aquarium contains 3" of gravel and the bucket is filled to a depth of 11", how many buckets of water will it take to fill the aquarium? Explain your strategy and show all necessary calculations.